Power Laws in Solar Flares: Self-Organized Criticality or Turbulence?

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We study the time evolution of Solar Flares activity by looking at the statistics of quiescent times τ_L between successive bursts. The analysis of 20 years of data reveals a power law distribution with exponent $\alpha \simeq 2.4$ which is an indication of complex dynamics with long correlation times. The observed scaling behavior is in contradiction with the Self-Organized Criticality models of Solar Flares which predict Poisson-like statistics. Chaotic models, including the destabilization of the laminar phases and subsequent restabilization due to nonlinear dynamics, are able to reproduce the power law for the quiescent times. In the case of the more realistic Shell Model of MHD turbulence we are able to reproduce all the observed distributions.

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Solar flares are sudden, transient energy release above active regions of the sun [1]. Energy is released in various form (thermal soft X–ray emission, accelerated particles, hard X–ray (HXR) emission, and so on). Parker [2] conjectured that flares represent the dissipation at the many tangential discontinuities arising spontaneously in the bipolar fields of the active regions of the Sun as a consequence of random continuous motion of the footpoints of the field in the photospheric convection [2]. Probability distributions, calculated for various observed quantities, x, can be well represented by power laws of the form $P(x) = Ax^{-\alpha}$. In particular from HXR emission, the distribution of peak flux yields $\alpha \simeq 1.7$, that of total energy associated with a single event yields $\alpha \simeq 1.5$ and finally the distribution of flare duration yields $\alpha \simeq 2$ [3].

The conjecture by Parker and the power laws found in the distributions of real flares stimulated a new way of looking at impulsive events like flares. In fact Lu and Hamilton [4] pointed out that Self–Organized Criticality (SOC), introduced earlier [5], could describe the main features of HXR flares, [6,7], even if recent 2D MHD simulations have been devoted to recover power laws in the energy dissipation [8]. What is usually called SOC is a mechanism of charging and discharging, apparently without tuning parameters, which reproduces self–similarity in critical phenomena.

Lu and Hamilton, [4], assume that the coronal magnetic field evolves in a self–organized critical state in which an "event" can give rise to other similar "events" through an avalanche process (sandpile model). An active region on the sun is thus modeled through a magnetic field $\boldsymbol{B}$ on a uniform 3D lattice. In order to have a statistically stationary state, energy is injected into the system by adding a small magnetic field increment $\delta\boldsymbol{B}$ at a random site on the grid. When an avalanche takes place, the energy input is suspended until all sites become stable. In this sense the avalanches (flares) are fast phenomena, on a time scale much smaller than the injection mechanism. The large popularity of these models settles on their capability to reproduce the power law behavior in the distribution functions of the total energy, the peak luminosity and the duration of avalanches.

What we want to stress in this letter is the fact that a different kind of statistics can be studied on solar flare signals: the distribution of laminar or waiting times, i.e. the time intervals between two successive bursts. This distribution has been recently studied on solar flares HXR events [9,10]. Wheatland et al. [10] have also emphasized the fact that this kind of distribution is crucial from the point of view of the avalanche model. SOC models indeed are expected to display an exponential waiting time distribution $P(\tau_L) = \langle\tau_L\rangle^{-1}\exp(-\tau_L/\langle\tau_L\rangle)$, where $\langle\tau_L\rangle$, the average laminar time, depends on the parameters of the model. This behavior is related to the fact that the avalanche duration is much smaller than the charging time (the time between two successive throws of magnetic field in random position) and charging place is independent on the avalanche position. Then one expects no correlation between successive bursts and thus a trivial statistics for the laminar times. This is clearly observed in a simulation of the SOC automaton that we have done (see the inset of figure 1). On the contrary all authors [9,10] found a more or less well defined power law distribution. In particular Wheatland et al. [10], by performing a careful statistical analysis of waiting time distribution on 8 years of solar flares HXR bursts observed by the ICE/ISEE 3 spacecraft, have shown that the distribution in no way can be attributed to a nonsta-

tionary Poisson process.

We have done the same statistics on laminar phases using twenty years of data from National Geophysical Data Center of USA. In this database starting and ending times as well as peak times of HXR bursts associated to flares from 1976 up to 1996, measured at the Earth by satellites in the 0.1 to 0.8 nm band, are stored. We calculated the laminar times as the time differences between two successive maxima of the flares intensity recorded during periods of activity of the same instrument. Two different kind of analysis have been performed: in the first one we have built up a dataset of about 1100 samples (hence on dataset A) by calculating only the differences between the time of occurrence of flares within the same active region, as identified through the Hα flares occurrence. To build up the second dataset (dataset B), we have considered the sun as a unique physical system, and we have calculated the time differences between two successive maxima of flare intensity regardless of the position of the flare on the sun surface. In this way we get a dataset of about $32,000$ samples. The analysis of these data shows that, in both cases, laminar times display a clear power–law distribution $P(\tau_L) = A\tau_L^{-\alpha}$ with $\alpha = 2.38 \pm 0.03$ in the range 6 hr $\leq \tau_L \leq$ 67 hr (reduced $\chi^2 = 2.2$) for dataset B and $\alpha = 2.4 \pm 0.1$ (reduced $\chi^2 = 1.1$) for dataset A(figure 1). The exact value of the exponent can be affected by the finite lenght of the observation times, which underestimates the occurrence of long waiting times. However these results, as well as those obtained by previous authors [9,10], allow us to consider the power law distribution of waiting times as firmly established as the power laws observed for total energy, peak luminosity and time duration and force us to investigate whether models different from SOC can account for all these distributions.

One point of the SOC philosophy is that the system has to be at the edge of chaos in order to display power laws [11]. Indeed in a chaotic system there exists a characteristic time t_c given in terms of the leading Lyapunov exponent as $t_c \simeq 1/\lambda$. This seems to be incompatible with the existence of self-similar scaling laws (i.e. no characteristic times) a part the limit case $\lambda = 0$. The above argument in not very strong. As a matter of fact we recall that it is possible to have chaotic systems showing scaling behavior in presence of many time scales [12] or in presence of strong fluctuations of the local Lyapunov exponent [13]. Thus it is worth investigating which kind of chaotic systems can give rise to self-similar scaling laws.

It is worth noting that the occurrence of a power law in the distribution of the laminar times is the analogous for the Solar flares of the Omori's law for the earthquakes [14] and represents a clear indication of the existence, in the flare dynamics of strong correlations between successive bursts, at variance with the SOC model. The unique possible origin of the correlations arises from non trivial evolution equations of the phenomenon. Because solar flares are governed by MHD equations, it is rather natural to investigate their statistics in the context of MHD turbulence. Turbulence is a common phenomenon in fluids, where chaotic dynamics and power law statistics coexist. Moreover fluid turbulence displays time intermittency, in that dissipative events are not uniformly but burstly distributed in time.

Before introducing the turbulence model, let us stress the fact that the intermittent behavior can be observed also in simple dynamical models. Let us consider the one dimensional random map $x_{t+1} = r_t x_t(1 - x_t)$, where at each step t the random variable r_t is extracted according to a given distribution, e.g. $r_t = 4$ with probability p and $r_t = 1/2$ with probability $1 - p$. If $p < 1/3$ $x = 0$ is an attracting fixed point. If $p = 1/3 + \delta p$ one has a rather interesting behavior called on-off intermittency [15]: x_t remains close to zero for a certain time (laminar time) then there is a short interval of strong activity (burst) after which a new quiescent phase takes place. Also in this simple model, if δp is not too large, one observes power law not only for the energy distribution (here defined as the integral of x_t over the burst), but also for the laminar times τ_L. The exponent of power law for τ_L turns out to be $\alpha = 1.5$ [16]. In spite of its simplicity, the random map model contains some basic ingredients of the relevant features of the time intermittency in dynamical systems, i.e. the destabilization of the laminar phase by linear instability and the subsequent restabilization due to the nonlinear dynamics.

Dynamical systems more directly related to fluid turbulence are the so called shell models [17]. Shell models represent a zero-order approximation of fluid equations (Navier-Stokes or MHD equations) in which one consider a single (complex) scalar variable u_n (and b_n) as representative of the velocity (magnetic) fluctuation associated to a wavenumber $k_n = k_0 2^n$ $(n = 1, ..., N)$. The fact that the wavenumbers k_n are exponentially spaced allows to reach very large Reynolds numbers with a moderate number of degrees of freedom and then to investigate regimes of $3D$ MHD turbulence which are not accessible by direct numerical simulation. The philosophy underlying the shell model approach to turbulence is that even with a relatively small dynamical system it is possible to reproduce some statistical features of the turbulent cascade. In particular shell models mimic at the best the time intermittency of real turbulent fluid flows.

The evolution equations for the dynamical variables u_n and b_n are built up by ignoring any detail of the spatial structure and boundary conditions. Only the interactions between nearest and next nearest neighbor shells are retained in the form of quadratic nonlinearities. The coupling coefficients of nonlinear terms are determined by imposing the inviscid conservation of the MHD quadratic invariants [17,18]. The particular shell model we used in our simulation reads [19]

$$\frac{du_n}{dt} = -\nu k_n^2 u_n + f_n + ik_n \left\{ (u_{n+1}u_{n+2} - b_{n+1}b_{n+2}) - \frac{1}{4}(u_{n-1}u_{n+1} - b_{n-1}b_{n+1}) - \frac{1}{8}(u_{n-2}u_{n-1} - b_{n-2}b_{n-1}) \right\}^*$$

$$\frac{db_n}{dt} = -\eta k_n^2 b_n + ik_n(1/6)\left\{(u_{n+1}b_{n+2} - b_{n+1}u_{n+2}) + (u_{n-1}b_{n+1} - b_{n-1}u_{n+1}) + (u_{n-2}b_{n-1} - b_{n-2}u_{n-1})\right\}^*$$

where ν and η are respectively the viscosity and the resistivity and f_n is an external forcing term acting only on velocity fluctuations.

Shell models are good models of turbulent cascade in the sense that they display, in the limit of fully developed turbulence $\nu, \eta \to 0$, scaling laws for the structure functions $S_p(n) = \langle |x_n|^p \rangle \sim k_n^{-\zeta_p}$ where x_n is either u_n or b_n. In the hydrodynamic limit ($b_n = 0$) the set of scaling exponents ζ_p are found to be very close to those obtained by experiments [18]. One indeed observes a clear deviation from the Kolmogorov scaling ($\zeta_p = p/3$) as a consequence of the intermittent dynamics of the system.

Another observable whose statistics is well reproduced in shell models is the energy dissipation $\epsilon(t)$ defined as $\epsilon(t) = \nu \sum_{n=1}^{N} k_n^2 |u_n|^2 + \eta \sum_{n=1}^{N} k_n^2 |b_n|^2$ which displays the characteristic intermittency of fully developed turbulence (see figure 2).

We have performed on $\epsilon(t)$ the same statistical analysis done for the solar flare signal. We define a burst of dissipation (corresponding to a flare) by the condition $\epsilon(t) \geq \epsilon_c$. This definition allows us to calculate the distribution functions for the peak values of the bursts, their total energy (defined as the integral of the signal above ϵ_c) and the duration of the bursts (defined as the time during which the dissipation is above ϵ_c). We have chosen the threshold as $\epsilon_c = \langle\, \epsilon(t)\, \rangle + 2\sigma$, where the average and the standard deviation have been calculated on the time intervals in between the bursts, through an iterative process in order to take into account only the background contribution. The results of the analysis are shown in figure 3. Also in this case we observe clear power law distribution functions with exponents $\alpha \simeq 2.05$ for the peak distribution, $\alpha \simeq 1.8$ for the total energy distribution and $\alpha \simeq 2.2$ for the burst durations. The exponents are close to those obtained in analyzing solar flares data, but we do not think that the agreement is particularly significant as the model exponents depend on the value chosen for the threshold. The relevant point is that, at variance with SOC models, MHD shell models display a power law statistics also for the laminar times, as shown in figure 4. The scaling exponent turns out to be $\alpha \simeq 2.70$, close to the one obtained from the experimental data.

The different behavior of SOC models and turbulent MHD shell models is related to the conceptually different mechanisms underlying the SOC phenomenon and the phenomenon of intermittency in fully developed turbulence. SOC models represent self-similar phenomena, while the intermittent behavior of turbulence is related to its chaotic nature [18]. Let us stress again that the actual value of the numerical scaling exponent is not important as it could depend on the details of the model. The statistics of the laminar times between two bursts is due to global properties of the system and it is thus more relevant than the properties of the single burst. In this sense, a good model for the flare bursts should be able to reproduce the quiescent time distributions. SOC models predict a Poissonian statistics for the scaling behavior of the quiescent times in the Solar flares activity. On the contrary chaotic models are able to reproduce the power law for the quiescent time since they include the correct mechanism for destabilization of the laminar phases and subsequent nonlinear restabilization.

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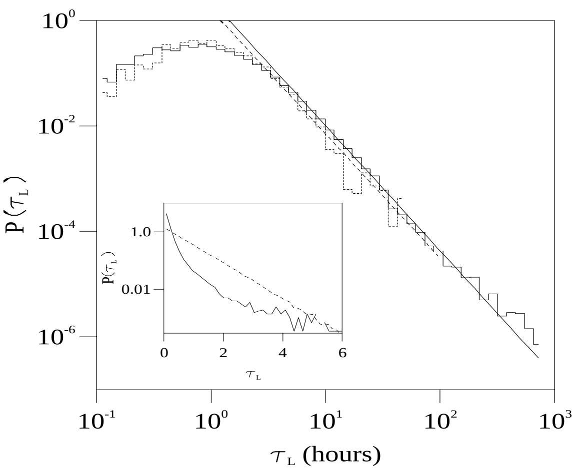

FIG. 1. Probability distribution of the laminar time $P(\tau_L)$ between two X–ray flares for dataset A (dashed line) and dataset B (full line). The straight lines are the respective power law fits. In the inset we show, in lin–log scale, the distribution for dataset B (full line) and the distribution obtained through the SOC model (dashed line) which displays a clear exponential law. The variables shown in the inset have been normalized to the respective root–mean–square values.

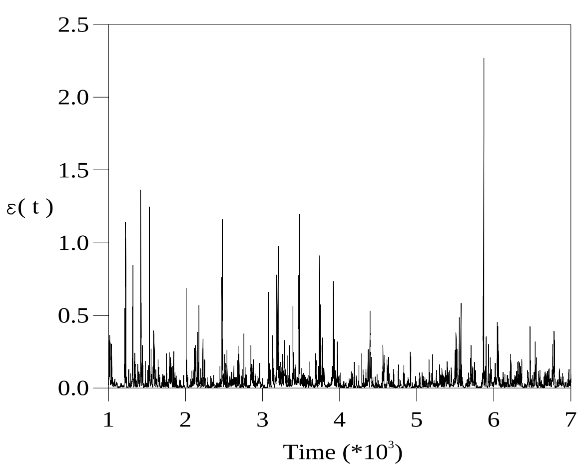

FIG. 2. Time series of energy dissipation $\epsilon(t)$ for the shell model. The parameters used in the simulation are $N = 19$, $\nu = \eta = 10^{-7}$, $k_0 = 1$. The external forcing term f_n is a stochastic variable acting only on the first two shells of the velocity fluctuations. It is calculated according to the Langevin equation $df_n/dt = -f_n/\tau_0 + \mu$, where τ_0 is the characteristic time of the largest shells ($\tau_0 \simeq 10$ in our units) and μ is a Gaussian white–noise with $\sigma = 0.1$.

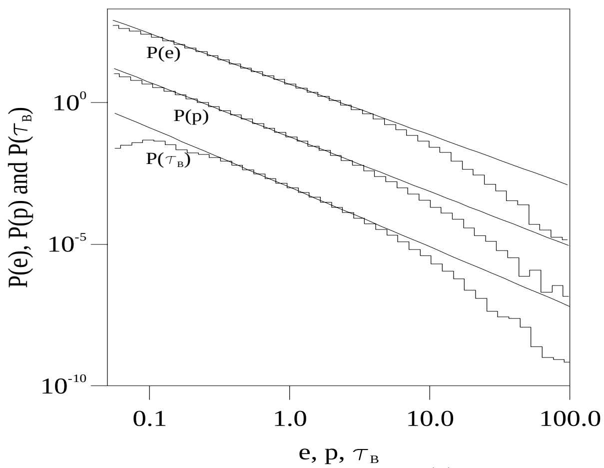

FIG. 3. Total energy distribution $P(e)$, energy peak distribution $P(p)$ and bursts duration distribution $P(\tau_B)$ for the shell model. The variables have been normalized to the respective root–mean–square values. The straight lines are the fits with power laws. The values of $P(e)$ and $P(\tau_B)$ are offset by a factor 100 and 10^{-2} respectively.

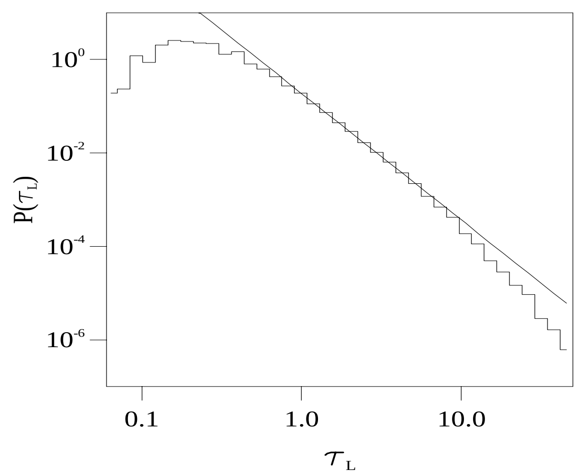

FIG. 4. The distribution of laminar times $P(\tau_L)$ for the Shell model, normalized to the root–mean–square. The straight line is the fit with a power law.